Elliptically polarized wave induced local growth of instability at an obliquely developed inhomogeneity in a plasma

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A strong elliptically polarized wave, incident obliquely on a plasma, consisting of singly charged ions and free electrons, when obstructed by an inhomogeneous growth at a point P (say) of its path of propagation, becomes unstable. When the characteristic length of variation of the inhomogeneity is of the order of micrometers, the wave energy is deposited completely in the vicinity of P. Plasma characteristics such as the wave-affected magnetic moment field, the wave growth factor, and the space charge separation wave of compression and rarefaction are also determined. The behavior of the resulting expanding shock front and the other effects cannot be considered by the field equations used here. Also the spin wave features (second-order effects) that develop and the related dispersion relations are considered for an elliptically polarized wave and a longitudinal wave. [S1063-651X(97)14703-1]

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I. INTRODUCTION

Consider a strong elliptically polarized wave (EPW) consisting of singly charged ions and free electrons, in a plasma facing an inhomogeneity at a field point P. The inhomogeneity is expressed as a finite gradient of the amplitude of the electric field of the wave at P, which is assumed to act parallel to the Z axis when the direction of wave propagation is in the Z-X plane and at an angle α with OX as in Fig. 1. The gradient action is weak when the characteristic length of variation of the field amplitude is large compared to the wavelength of the wave field. In that case the wave escapes distorted through the region, and may not remain elliptically polarized. On the other hand, when the characteristic length of variation of the amplitude is small (of the order of its own wavelength, or even smaller), say a few micrometers, for optical waves, the action of the instability from the inhomogeneity will be severe and the EPW will be destroyed, so that the whole of the wave train energy will be deposited in the neighborhood of P. This instability will be studied in this paper. We mention here that this growth is not of ponderomotive origin because that is an average effect along a domain of lengths of the order of one wavelength.

Here we obtain the local dispersion relation for the EPW at P, in terms of the amplitudes a and b and their partial derivatives with respect to z. The wave vector $\mathbf{K}$ is at an angle α with OX and the plane of incidence is the X-Z plane (Fig. 1). The amplitudes $a(z)$ and $b(z)$ of the electric field of the wave are slowly varying functions of z. Derivative of a and b with respect to z are denoted by a' and b'. We also retain a'' and b'' (derivative of a' and b') because the a' and b' terms may cancel out in some places. Both a', b', and a'', b'' are regarded as constant parameters. Finally a' is replaced by a/l, b', b/l, a'', a/r^2, and b'' by b/r^2 where l is the characteristic length of variation of a and b and r is that of a' and b'. To eliminate the complications from steepest descent in the profile of ''a'' and ''b'', we also assume that $l<r$. This is a simple way to estimate the growth of the inhomogeneity quantitatively, which destabilizes the wave. The influence of the amplitude gradient from the Gaussian law for the electric displacement vector can be studied independently of that from the other laws. So, to distinguish between these influences, in the Gauss equation we replace a', b', a'', and b'' by a_1', b_1', a_1'', and b_1'', respectively, write $a_1'=a_1/l_1$, $b_1'=b_1/l_1$, replace a_1'' by a_1/r_1^2 and b_1'' by b_1/r_1^2, and finally put $a_1=a$, $b_1=b$, but retain l_1 and r_1 as distinct from l and r. This does not create any problem in the calculations and in drawing conclusions. The space charge wave is essentially of compressibility and rarefaction and excites a longitudinal acoustic wave that does not exist for incident transverse waves in homogeneous plasma.

Due to incidence of strong circularly polarized waves (CPW) on material targets the electrons are driven in circular orbits generating (both dc and ac) fields of magnetic moment, also called the inverse Faraday effect (IFE). Including the relativistic variation of mass, for electron motion, the IFE was studied by Steiger and Woods [2] in unmagnetized plasma. A nonrelativistic treatment of IFE by a microwave radiation was given by Pomeau and Quemada [3] in a collisionless plasma. Deschamps *et al.* [4] observed this magnetic field in plasma experimentally. In some problems of wave-

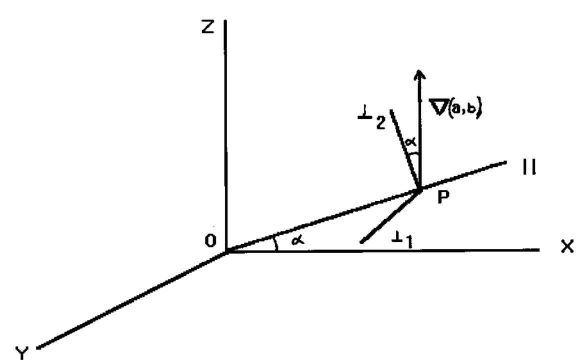

FIG. 1. Shows the direction of propagation of an elliptically polarized wave, the gradients of the amplitudes (a,b) of which act parallel to OZ at P.

plasma interaction and wave-wave interaction including the resonant and parametric interactions in plasma, the nonrelativistic dc part of the field is evaluated [5–8]. In the interaction of intense short wave field pulses [9,10], obliquely incident on a material target, the spatial gradient and nonstationary character of the ponderomotive force can generate [11,12] very large transverse magnetic field (of the order of 10^9 G) and a moderate axial magnetic field [13] (0.5–0.6 MG). Some amplitude-dependent instability processes are believed to generate the field in the coronal region [12] of the field produced plasmas. In the far-coronal region, the long-wavelength approximation $\lambda \gg \lambda_D$ (λ is the wavelength of the applied wave field and λ_D is the Debye wavelength) is not valid. There the density steepening effect gives rise to filamentation [14], the conditions $L_N > \lambda$ and $L_E > \lambda$ are violated, and the space charge separation effect [11] is possible when $L_N < \lambda < \lambda_D$ and $L_E < \lambda < \lambda_D$ where L_N and L_E are the characteristic lengths of variation of density and the amplitude of the electric field, respectively. These features and the spin wave features are studied here using the magnetic moment evolution equation [15].

Magnetization in plasma is a direct instantaneous process of conversion of energy available in other forms into magnetization energy. The permanent magnetization in a magnetic material below the Curie temperature T_C is explained by the concepts of the spin angular momentum distribution of electrons. The permanent magnetization of Earth, Sun, etc., which are made up of plasma, should be explained by the theory of permanent magnetization in magnetic material. Then it should be remembered that plasmas do not require the latent heat of phase transition and have no thermal limit of Curie temperature T_C of magnetization, because the plasma collective behavior is not destroyed at high temperature but that of the other magnetic materials is destroyed. The effects of induced electric dipole moments are included in the dielectric tensor $\boldsymbol{\epsilon}(\omega,K)$ and the magnetic dipoles in the magnetic permeability tensor $\boldsymbol{\mu}(\omega,K)$. The theories of their determination from the constitutive relations are independent of the derivation of the macroscopic Maxwell equations.

The second- and higher-order elements μ_{ij} of $\boldsymbol{\mu}$ depend on the parameters of the plasma and the applied wave field, and excited spin waves. As in solid state plasmas, the elements ϵ_{ij} of $\boldsymbol{\epsilon}$ are associated with electrokinetic waves and μ_{ij} of $\boldsymbol{\mu}$ with spin waves; so both $\boldsymbol{\epsilon}$ and $\boldsymbol{\mu}$ should exist. Then the collective effects allow the excitation of the two categories of waves, and their study classically in media including plasmas.

Magnetization in a magnetic material is explained quantum mechanically by the action of spin waves. But their classical study in plasma as second- and higher-order effects is permitted in some regions of wave frequency ω and wave number K, with the help of the magnetic moment evolution equation

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}^0 \times \mathbf{H}), \quad \text{where} \quad \gamma = |e|/2m_0c,$$

which is derived with the help of classical Newtonian mechanics. The elements of the μ_{ij} are calculated with the help of this equation [15], where the zero frequency magnetic moment field is determined by Eq. (19) in terms of the parameters of the applied wave field and the medium.

So the elements of μ_{ij} and ϵ_{ij} are of second and higher order and are obtained as functions of the parameters of the wave field and the medium. Then, finally, spin wave features including the dispersion features are determined by feeding the phenomenological Maxwell equations of electrodynamics with the nonzero elements of μ_{ij} and ϵ_{ij}, and doing some standard analysis.

II. FORMULATION OF THE PROBLEM

A. Basic equations and dispersion relation

The basic Maxwell equations of a Lorentz model for cold, nonrelativistic, collision-free, unmagnetized electron plasma are

$$\nabla \times \mathbf{E} = -\frac{1}{C}\frac{\partial \mathbf{H}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{H} = \frac{1}{C}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{C}\mathbf{J}, \tag{2}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho^c, \tag{3}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{4}$$

where $\rho^c = \Sigma_{\beta=e,i} N_\beta q_\beta$ and $\mathbf{J} = \Sigma_{\beta=e,i} N_\beta q_\beta v_\beta$ are the charge and the current density, respectively. The electrons and ions have charge $q_e(=-e)$ and $q_i(=e)$ per particle, number densities N_e, N_i, and velocities $\mathbf{v}_e$, $\mathbf{v}_i$, respectively.

The EPW (Fig. 1) has the electric field vector

$$\mathbf{E} = [a(z)\cos\theta, b(z)\csc\alpha \sin\theta, -a(z)\cot\alpha \cos\theta] \tag{5}$$

where $\theta = (\mathbf{k}\cdot\mathbf{r} - wt) = (kx \cos\alpha + kz \sin\alpha - wt)$, the $a(z)$ and $b(z)$ field amplitudes are slowly varying functions of z, and the z-x plane is the plane of incidence. Ions remain static and provide the background for macroscopic charge neutrality, which is violated by the action of the inhomogeneity at P. Taking divergence of $\mathbf{E}$, the space charge separation effect is obtained

$$\nabla \cdot \mathbf{E} = -a_1'(z)\cot\alpha \cos\theta = 4\pi\rho^c = -4\pi N_e, \tag{6}$$

where ρ^c (the space charge density) $= -eN$. The pressure variation ∇p corresponding to the electron number density fluctuation N induced by the wave interaction with the inhomogeneity in plasma at P is obtained from the adiabatic law of compressibility, given by

$$\nabla p = C_s^2 \nabla \rho^{\mathrm{m}} = mC_s^2 \nabla N = \frac{mC_s^2}{4\pi e}\nabla(a_1'\cot\alpha \cos\theta), \tag{7}$$

where Eq. (6) is used. Here ρ^{m} is the perturbation in the electron mass density, and C_s is the velocity of sound in the electron fluid. This space charge induced pressure force is included as a force in addition to the applied force of the electromagnetic (em) wave in the equation of motion, which, in the linearized approximation, reads

$$mN_0\dot{\mathbf{v}} = -\nabla p - eN_0\mathbf{E} \tag{8}$$

Solving for $\dot{\mathbf{v}}$ from this equation we obtain

$$\dot{\mathbf{v}}=-\frac{C_s^2}{4\pi N_0 e}[\hat{\mathbf{i}}(-a_1' K\cot\alpha\cos\alpha\sin\theta)+\hat{\mathbf{k}}(-a_1' K\cos\alpha\sin\theta+a_1''\cot\alpha\cos\theta)]-\frac{e}{m}[\hat{\mathbf{i}}(a\cos\theta)+\hat{\mathbf{j}}b\csc\alpha\sin\theta+\hat{\mathbf{k}}(-a\cot\alpha\cos\theta)],$$

where $\hat{\mathbf{i}},\hat{\mathbf{j}},\hat{\mathbf{k}}$ are unit vectors along the directions of coordinate axes OX, OY, and OZ, respectively. The time integral of this equation gives

$$v_x^{(1)}=\alpha_e C\left\{\frac{n_s^2}{XKl_1}\cot\alpha\cos\alpha\cos\theta+\sin\theta\right\},$$
$$v_y^{(1)}=\beta_e C(-\csc\alpha\cos\theta), \tag{9}$$
$$v_z^{(1)}=\alpha_e C\left\{\frac{n_s^2}{XKl_1}\cos\alpha\cos\theta+\left(\frac{n_s^2}{XK^2r_1^2}-1\right)\cot\alpha\sin\theta\right\},$$

where $\alpha_e=ea/m\omega c$, $\beta_e=eb/m\omega c$, $n_s=KC_s/\omega$, $X=\omega_p^2/\omega^2$, $\omega_p^2=4\pi N_0 e^2/m$, and $a'=a/l_1$, $a''=a/r_1^2$.

So α_e,β_e are dimensionless parameters (amplitude-like) of the wave field and n_s is a refractive indexlike parameter.

Since $\mathbf{J}=-N_0 e\mathbf{v}$, the Maxwell equations in the linearized approximation give

$$\nabla(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}+\frac{1}{C^2}\ddot{\mathbf{E}}=-\frac{4\pi}{C^2}\dot{\mathbf{J}}=\frac{4\pi N_0 e}{C^2}\dot{\mathbf{v}}. \tag{10}$$

Solving Eq. (10) for finding the locally valid dispersion relations generated by the action of the inhomogeneity on the EPW, we obtain

$$v_x^{(2)}=\frac{\alpha_e C}{X}\left\{\left(1-n^2+\frac{n^2}{K^2r^2}\right)\sin\theta+\frac{2n^2}{Kl}\sin\alpha\cos\theta+\frac{n^2}{Kl_1}\cot\alpha\cos\alpha\cos\theta\right\},$$

$$v_y^{(2)}=\frac{\beta_e C}{X}\left\{-\left(1-n^2+\frac{n^2}{K^2r^2}\right)\csc\alpha\cos\theta-\frac{2n^2}{Kl}\sin\theta\right\},$$

$$v_z^{(2)}=\frac{\alpha_e C}{X}\left\{-\left(1-n^2+\frac{n^2}{K^2r^2}\right)\cot\alpha\sin\theta-\frac{2n^2}{Kl}\cos\alpha\times\cos\theta+\frac{n^2}{Kl_1}\cos\alpha\cos\theta+\frac{n^2}{K^2r_1^2}\cot\alpha\sin\theta\right\}, \tag{11}$$

where $n=[KC/\omega]$ is the plasma refractive index. The term a_1' effectively appears in the two components in the plane of incidence and a' appears in all three components. To obtain the dispersion relation, it is better to equate the components of velocity from Eqs. (9) and (11), after transforming them at P into components parallel to the wave vector $\mathbf{K}$, denoted by the subscript $\parallel$ and having the direction cosines $(\cos\alpha,0,\sin\alpha)$, perpendicular to $\mathbf{K}$ in the plane of incidence, denoted by the subscript $\perp_2$ and having the direction cosines $(-\sin\alpha,0,\cos\alpha)$, and perpendicular to the plane of incidence, denoted by the subscript $\perp_1$ with direction cosines (0,1,0) which coincide with the y direction. Thus

$$v_\parallel=v_x\cos\alpha+v_z\sin\alpha,$$
$$v_{\perp_1}=v_y,$$
$$v_{\perp_2}=-v_x\sin\alpha+v_z\cos\alpha. \tag{12}$$

B. Local dispersion relations

Using Eq. (12), transformation of Eqs. (9) and (11) to the new coordinate system generates equations that can be written as

$$A_\parallel^1\cos\theta=A_\parallel^2\sin\theta,$$
$$A_{\perp_1}^1\cos\theta=A_{\perp_1}^2\sin\theta,$$
$$A_{\perp_2}^1\cos\theta=A_{\perp_2}^2\sin\theta.$$

Eliminating θ from these relations we obtain two local dispersion relations. One of these is for the coupled longitudinal wave

$$\left(1-n^2-X+\frac{n^2}{K^2r^2}\right)=-\frac{2n^2r_1^2}{ll_1}, \tag{13}$$

and the other for the transverse waves is

$$\left(1-n^2-X+\frac{n^2}{K^2r^2}-\frac{n^2}{K^2r_1^2}\right)\left(1-n^2-X+\frac{n^2}{K^2r^2}\right)=-\frac{4n^4}{K^2l^2}\sin^2\alpha, \tag{14}$$

where for simplification we have used the fact that $n^2\gg n_s^2$.

The dispersion relation for the longitudinal wave, valid in the neighborhood of the point P, can be written as

$$\omega^2-\omega_p^2+\frac{C^2}{r^2}=(1-2\delta^2)K^2c^2, \tag{15}$$

where $\delta^2=r_1^2/ll_1$ is a dimensionless parameter. It gives complex values for ω^2 because the dimensionless quantity $2\delta^2>1$ for slow variation of the amplitudes without change of curvature. To avoid the complications of steep descent at P we therefore assume $r_1>l_1$. For astrophysical plasmas, r,r_1,l,l_1 are large distances, sometimes even of the order of kilometers, so when $2\delta^2>1$, the wave cannot propagate beyond P.

There are two cutoff frequencies of the transverse waves, ω_{C_1} and ω_{C_2}; these are given by

$$\omega_{C_1}^2=\omega_p^2-\frac{C^2}{r^2}+\frac{C^2}{r_1^2}, \tag{16}$$

$$\omega_{C_2}^2=\omega_p^2-\frac{C^2}{r^2}. \tag{17}$$

Evidently $\omega_{C_1} > \omega_{C_2}$, and since $\omega_{C_2} < \omega_p$, the wave cannot penetrate up to the regions it can in the absence of the obstacle at P. In space plasmas ω_{C_1} and ω_{C_2} are slightly different from ω_p.

In laser induced plasmas, r is of the order of a few micrometers and $\omega_p \approx 10^{15}$/sec; so C/r is larger than ω_p, and instability will set in near P. When C/r and C/r_1 are small compared to the wave frequency ω, the dispersion relation (14) becomes

$$(\omega^2 - K^2C^2 - \omega_p^2)^2 = -\frac{4K^2C^4}{l^2\omega^4}\sin^2\alpha. \tag{18}$$

The roots are imaginary even for $\omega^2 > \omega_c^2$, so instability will set in.

C. Zero harmonic induced magnetization

The formula for the magnetic moment, per unit volume, at a point $\mathbf{r}$ at time t, is

$$\mathbf{M} = \frac{1}{2C}(\boldsymbol{\xi}\times\mathbf{j}) = -\frac{eN_0}{2C}(\boldsymbol{\xi}\times\mathbf{v}), \tag{19}$$

where $\boldsymbol{\xi}$ is the displacement of the charged particles and J is the current density. For the wave field (5), integrating Eq. (12) and using Eq. (19), we obtain the component of the dc (zero harmonic) magnetic moment density at P along the three orthogonal directions specified after Eq. (11),

$$M_{\parallel}^0 = \frac{\alpha_e\beta_e N_o e}{2\omega}\left\{\left(1 - \frac{n_s^2}{XK^2r_1^2}\right)\cot^2\alpha + 1\right\}. \tag{20}$$

$$M_{\perp_1}^0 = \frac{\alpha_e^2 e N_0}{2\omega}\left\{\frac{n_s^2}{XKl_1}\csc^2\alpha\,\cos\alpha\right\}, \tag{21}$$

$$M_{\perp_2}^0 = \frac{\alpha_e\beta_e e N_0}{2\omega}\left\{\frac{n_s^2}{XK^2r_1^2}\cot\alpha\right\}. \tag{22}$$

Evidently, this is a primary source of generation of the dc magnetic moment and there will be both axial and lateral dc magnetizations. In the tube geometry, this is a primary source of generation of both poloidal and toroidal dc fields. The dispersion relations indicate the growth of noncollisional instability due to the presence of the inhomogeneity at P. This will amplify the magnetic field, and, within times of the order of the period of the second harmonic of the applied field, instability will set in, making further study by this theory invalid. Hence, for weakly but continuously varying layers parallel to the ZX plane, the elliptically polarized wave can excite axial (poloidal) as well as radial (toroidal) dc magnetization. This action of the inhomogeneity contributes to the noncollisional dissipation of energy. Specifically, for a wave of high intensity and long wavelength, proceeding towards the coronal region, this noncollisional dissipation of the dc magnetic moment is dominant.

D. The second order magnetic permeability

Using Eq. (1) the components of the dispersive magnetic field H_x, H_y, and H_z are calculated and using a relation similar to Eq. (12) for H we obtain

$$H_{\parallel} = 0,$$

$$H_{\perp 1} = \frac{a}{\omega}\left(\frac{\sin\theta}{l_1} + K\csc\alpha\,\cos\theta\right), \tag{23}$$

$$H_{\perp_2} = \frac{Kb}{\omega}\csc\alpha\,\sin\theta.$$

Equations (20)–(23) and the relation

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}^0\times\mathbf{H}) \tag{24}$$

determine the components of the magnetic moment density $\mathbf{M}$ along the three local orthogonal directions at P.

$$M_{\parallel} = -\frac{\gamma\alpha_e^2\beta_e N_0 C^2 m}{2\omega^2}\frac{n_s^2}{X}\cot\alpha\left[\left(\frac{\csc^2\alpha}{l_1} - \frac{1}{K^2r_1^2l_1}\right)\times\cos\theta - \frac{1}{K^2r^2}\csc\alpha\,\sin\theta\right], \tag{25}$$

$$M_{\perp_1} = -\frac{\gamma\alpha_e\beta_e^2C^2N_0mk}{2\omega^2}\times\left\{\left(\frac{n_s^2}{XK^2r_1^2} - 1\right)\cot^2\alpha + 1\right\}\csc\alpha\,\cos\theta, \tag{26}$$

$$M_{\perp_2} = -\frac{\gamma\alpha_e^2\beta_e C^2N_0m}{2\omega^2}\left\{\left(\frac{n_s^2}{XK^2r_1^2} - 1\right)\cot^2\alpha + 1\right\}\times\left(\frac{\cos\theta}{l_1} - K\csc\alpha\,\sin\theta\right). \tag{27}$$

Also, the tensor relations

$$\mathbf{B} = \boldsymbol{\mu}:\mathbf{H} = \mathbf{H} + 4\pi\mathbf{M}$$

give

$$\mu_{12} = -\frac{4\pi\gamma\alpha_e^2\beta_e C^2N_0m}{2aK\omega}\frac{n_s^2}{Xl_1}\left(\cot^2\alpha - \frac{\cos\alpha}{K^2r_1^2}\right), \tag{28}$$

$$\mu_{13} = -\frac{4\pi\gamma\alpha_e^2\beta_e C^2N_0m}{2\omega b}\frac{n_s^2}{XK}\left\{\frac{\cos\alpha\,\csc^2\alpha}{r_1^2} + \frac{1}{l^2}\left(\cot^2\alpha - \frac{\cos\alpha}{K^2r_1^2}\right)\right\}\sin\alpha \tag{29}$$

$$\mu_{22} = 1 - \frac{4\pi\gamma\alpha_e\beta_e^2C^2N_0m}{2a\omega}\times\left\{\left(\frac{n_s^2}{XK^2r_1^2} - 1\right)\cot^2\alpha + 1\right\}\csc\alpha, \tag{30}$$

$$\mu_{23}=\frac{4\pi\gamma\alpha_e^2\beta_e C^2 N_0 m}{2Kl_1 b\omega}\left\{\left(\frac{n_s^2}{XK^2r_1^2}-1\right)\cot^2\alpha+1\right\}, \tag{31}$$

$$\mu_{32}=-\frac{4\pi\gamma\alpha_e^2\beta_e C^2 N_0 m}{2aK\omega l_1}\left\{\left(\frac{n_s^2}{XK^2r_1^2}-1\right)\cot^2\alpha+1\right\}\sin\alpha, \tag{32}$$

$$\mu_{33}=1-\frac{4\pi\gamma\alpha_e^2\beta_e C^2 N_0 m}{2Kb\omega}\left[\frac{-1}{Kl_1^2}+\left\{\left(\frac{n_s^2}{XK^2r_1^2}-1\right)\times\cot^2\alpha+1\right\}K\csc\alpha\right]\sin\alpha, \tag{33}$$

$$\mu_{11}=\mu_{21}=\mu_{31}=0.$$

E. The electrical permittivity tensor

Since $E_\parallel=0$, $E_{\perp_1}=b\csc\alpha\sin\theta$ and $E_{\perp_2}=-a\csc\alpha\cos\theta$, we find that

$$\epsilon_{12}=\frac{a}{b}\frac{n_s^2}{Kl_1}\cos\alpha, \tag{34}$$

$$\epsilon_{13}=\frac{n_s^2}{K^2r_1^2}\cos\alpha, \tag{35}$$

$$\epsilon_{22}=(1-X), \tag{36}$$

$$\epsilon_{33}=(1-X)+\frac{n_s^2}{K^2r_1^2}, \tag{37}$$

$$\epsilon_{11}=\epsilon_{21}=\epsilon_{31}=\epsilon_{23}=\epsilon_{32}=0.$$

The Ampere-Maxwell law for the magnetoelectric induction and the relation $\mathbf{j}=-$neu, where $\dot{u}=-eE/m$, determines the elements of ϵ_{ij} in the linearized approximation, independent of the amplitudes "a" and "b" of the electric field.

F. Calculation of local spin wave dispersion characteristics

The obliquely incident em wave spoiled by the inhomogeneity makes the magnetic permeability a tensor in the second approximation. Hence, in the plasma medium the spin wave (essentially a transverse wave) is studied by the Maxwell equations

$$\nabla\times\mathbf{E}-\frac{\boldsymbol{\mu}:}{C}\frac{\partial\mathbf{H}}{\partial t},\quad \nabla\cdot\mathbf{D}=0,$$

$$\nabla\times\mathbf{H}=\frac{\boldsymbol{\epsilon}:}{C}\frac{\partial\mathbf{E}}{\partial t},\quad \nabla\cdot\mathbf{B}=0.$$

Taking the Curl of the first equation, we obtain

$$\nabla(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}=-\boldsymbol{\mu}:\boldsymbol{\epsilon}\ddot{\mathbf{E}}.$$

Hence,

$$\nabla^2\times\begin{bmatrix}0\\E_{\perp_1}\\E_{\perp_2}\end{bmatrix}=\frac{1}{C^2}\times\begin{bmatrix}0&\mu_{12}\epsilon_{22}&\mu_{13}\epsilon_{33}\\0&\mu_{12}\epsilon_{22}&\mu_{23}\epsilon_{33}\\0&\mu_{32}\epsilon_{22}&\mu_{33}\epsilon_{33}\end{bmatrix}\begin{bmatrix}0\\\ddot{E}_{\perp_1}\\\ddot{E}_{\perp_2}\end{bmatrix} \tag{38}$$

and the dispersion relations are

$$\frac{K^2C^2}{\omega^2}=\epsilon_{22}\left(\mu_{22}-\frac{\mu_{13}\mu_{12}}{\mu_{23}}\right) \tag{39}$$

and

$$\frac{K^2C^2}{\omega^2}=\epsilon_{33}\left(\mu_{33}-\frac{\mu_{13}\mu_{32}}{\mu_{12}}\right). \tag{40}$$

III. OUTPUTS FROM A LONGITUDINAL WAVE

For a longitudinal wave

$$E=\{a(z)\cos\alpha,0,a(z)\sin\alpha\}e^{i\theta}; \tag{41}$$

and using the same basic equations as before, the components of velocity, obtained from the equation of motion, are

$$v_x^{(1)}=\frac{n_s^2\alpha_e C_s}{XKl_1}\sin\alpha\cos\alpha e^{i\theta}-i\alpha_e C_s\left(1+\frac{n_s^2}{X}\right)\cos\alpha e^{i\theta}, \tag{42}$$

$$v_y^{(1)}=0,$$

$$v_z^{(1)}=\frac{n_s^2\alpha_e C_s}{XKl_1}(1+\sin^2\alpha)-i\alpha_e C_s\left(\frac{n_s^2}{XK^2r_1^2}+\frac{n_s^2}{X}+1\right)\times\sin\alpha e^{i\theta} \tag{43}$$

and those obtained from the Maxwell equations are

$$v_x^{(2)}=-\frac{\alpha_e c}{X}\left(i\frac{n^2}{K^2r^2}\cos\alpha-\frac{n^2}{Kl}\sin\alpha\cos\alpha+i\cos\alpha\right)e^{i\theta}, \tag{44}$$

$$v_y^{(2)}=0,$$

$$v_z^{(2)}=-\frac{\alpha_e C}{X}\left(\frac{n^2}{Kl}\cos^2\alpha+i\sin\alpha\right)e^{i\theta}. \tag{45}$$

Using the same procedure, the $\parallel$ and $\perp$ components of $v^{(1)}$ and $v^{(2)}$ are calculated; the dispersion relation is then

$$\frac{n^2}{K^2r_1^2}\cos^2\alpha=-1, \tag{46}$$

where the condition $C_s\ll C$ has been used. The cutoff frequency for the inhomogeneity-induced transverse wave is

$$\omega^2=\frac{C_s}{C}\left(\omega_p^2-\frac{C^2}{r_1^2}\right)\cos^2\alpha. \tag{47}$$

Using Eq. (19) the magnetic moment is found to be along the $\perp_1$ direction (y axis),

$$M_{\perp 1}=\frac{4\pi N_0 e}{2C\omega}\left[\frac{n_s^2\alpha_e^2 C_s^2}{XKl_1}\left\{\left(\frac{n_s^2}{XK^2r_1^2}+\frac{n_s^2}{X}+1\right)-\left(1+\frac{n_s^2}{X}\right)(1+\sin^2\alpha)\right\}\right]. \tag{48}$$

And $M_\parallel=0$, $M_{\perp_2}=0$ also, as $H_\parallel$ and $H_{\perp_2}$ both are zero, the excitation of the spin waves using equation (24) is not possible in this case [as the right hand side of Eq. (24) is zero].

IV. CONCLUSIONS

Spin waves are excited when a transverse, elliptically polarized wave comes in contact with a plasma inhomogeneity, which is represented by the gradient of the wave amplitude at an angle with the direction of the wave path. The nonzero elements of the tensor form of the magnetic permeability are determined here analytically. The general dispersion relations for transverse and longitudinal propagating waves, and two specific dispersion relations for the spin wave propagation for the transverse wave are obtained. Situations are found to be different for incident transverse and longitudinal waves. In the transverse case, spin wave excitations are possible, but for the longitudinal wave the spin wave excitation is not possible because $(M^0\times H)$ vanishes.

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